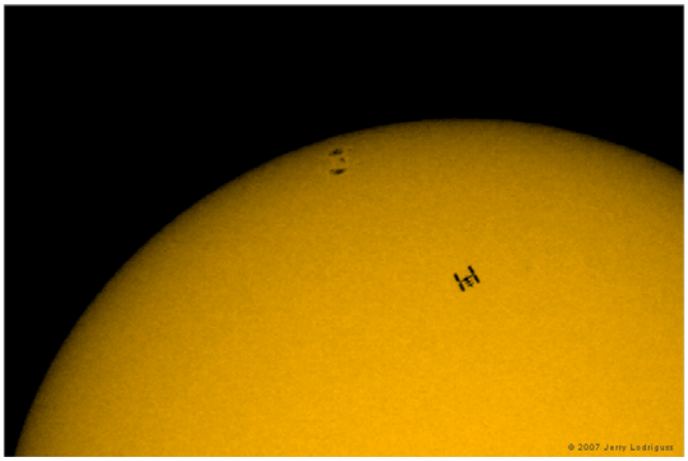
Angular Size and Velocity



The relationship between the distance to an object, **R**, the objects size, **L**, and the angle that it subtends at that distance, θ, is given by:

$$\theta = 57.29 \frac{L}{R} \text{ degrees}$$

$$\theta = 3{,}438 \frac{L}{R} \text{ arcminutes}$$

$$\theta = 206{,}265 \frac{L}{R} \text{ arcseconds}$$

To use these formulae, the units for length, L, and distance, R, must be identical.

Problem 1 - You spot your friend (L = 2 meters) at a distance of 100 meters. What is her angular size in arcminutes?

Problem 2 - The sun is located 150 million kilometers from Earth and has a radius of 696.000 kilometers, what is its angular diameter in arcminutes?

Problem 3 - How far away, in meters, would a dime (1 centimeter) have to be so that its angular size is exactly one arcsecond?

Problem 4 - The spectacular photo above was taken by Jerry Lodriguss (Copyright 2007, http://www.astropix.com/HTML/SHOW_DIG/055.HTM) and shows the International Space Station streaking across the disk of the sun. If the ISS was located 379 kilometers from the camera, and the ISS measured 73 meters across, what was its angular size in arcseconds?

Problem 5 - The orbital speed of the space station is 7.4 kilometers/second. If its distance traveled in 1 second is 7.4 kilometers, A) what was the angle, in arcminutes, that it moved through in one second as seen from the location of the camera? B) What was its angular speed in arcminutes/second?

Problem 6 - Given the diameter of the sun in arcminutes (Problem 2), and the ISS angular speed (Problem 5) how long, in seconds, did it take the ISS to travel across the face of the sun?

Extra for Experts: From the definition of the sine of an angle is given by the formula below, show that for small angles (X << 1), the three formulae above are obtained.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \ldots.$$

Answer Key

Problem 1 - Answer: Angle = 3,438 x (2 meters/100 meters) = 68.8 arcminutes.

Problem 2 - Answer: 3,438 x (696,000/150 million) = 15.9 arcminutes in radius, so the diameter is 2 x 15.9 = 31.8 arcminutes.

Problem 3 - Answer: From the second formula R = 3438 * L/A = 3438 * 1 cm/1 arcsecond so R = 3,438 centimeters or a distance of 34.4 meters.

Problem 4 - Answer: From the third formula, Angle = 206,265 * (73 meters/379,000 meters) = 39.7 arcseconds.

Problem 5 - Answer: The orbital speed of the space station is 7.4 kilometers/second. If its distance traveled in 1 second is 7.4 kilometers, A) The ISS traveled L = 7.4 kilometers so from the second formula Angle = 3,438 * (7.4 km/379 km) = 67 arcminutes. B) The angular speed is just 67 arcminutes per second.

Problem 6 - Answer: The time required is T = 31.8 arcminutes/ (67 arcminutes/sec) = 0.47 seconds.

Extra for Experts: From the definition of the sine of an angle is given by the formula below, show that for small angles, the three formulae above are obtained.

Note that as the angle measured in radians, x, becomes very small, only the first term remains significant, so sin (x) = x. Since the length, L, of a chord of a circle with a radius, R, is given by L = R sin(x), we have in the 'small angle approximation', L = R x ,or x = L/R where x is in radians. Since 1 radian = 57.29 degrees, the first formula follows. Since there are 60 arcminutes in 1 arcdegree, 57.2958 x 60 = 3,438 arcminutes in one radian and the second formula follows. Since there are 60 arcseconds in 1 arcminute, 3,438 x 60 = 206,265 arcseconds in one radian and the third formula follows.

The spectacular photo by Jerry Lodriguss had to be taken with careful planning beforehand. He had to know, to the second, when the sun and ISS would be in the right configuration in the sky as viewed from his exact geographic location. Here's an example of the photography considerations in his own words:

" I considered trying to monitor the transit visually with a remote release in my hand and just firing (the camera) when I saw the ISS in my guidescope. Then I worked out the numbers. I know that my reaction time is 0.19 seconds. This is actually quite good, but I make my living shooting sports where this is critical, so I better be good at it. I also know that the Canon 1D Mark IIn has a shutter lag of 55 milliseconds. Adding these together, plus a little bit of a fudge factor, the best I could hope for was about 1/4 of a second from when I saw it to when the shutter opened. Since the entire duration of the transit was only 1/2 of a second, in theory, I could capture the ISS at about the center of the disk if I fired as soon as I saw it start to cross. This was not much of a margin for error. I could easily blink and miss the whole thing... Out of 49 frames that the Mark IIn recorded, the ISS is visible in exactly one frame."